Permutation:

Permutation is selection and then arrangement of the selected items. Therefore it entails both operations. Therefore combination or selection is a subset of permutation.

The most common formulae is ${}^{n}P_{r}$

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

For example ${}^{10}P_2 = \frac{10!}{(10-2)!} = \frac{10!}{8!} = 9 \times 10 = 90$

So if *r* people have to be selected, from *n* & then arranged

$$\therefore \quad {}^{n}P_{r} = {}^{n}C_{r} \times r!$$

Say we have to form a 3 digit number from the digits 1, 2, 3, 4, 5

Now any of the 5 digits can in the 1st place: 5 x ? x ?

Any of the remaining 4 can go in the 2nd place: 5 x 4 x ?

Any of the remaining 3 can go in the 3rd place: 5 x 4 x 3

So total number of ways = 5 x 4 x 3

$5 \times 4 \times 3 = \frac{5\times4\times3\times2\times1}{2\times1} = \frac{5!}{2!} = \frac{5!}{(5-3)!} = {}^{5}P_3$

Solved Examples

1) There are 14 players in a squad. In how many ways 11 players can be decided with their respective batting order?

Answer: We have to select 11 players and then arrange them. This can be done in ${}^{14}P_{11}$ ways. ${}^{14}P_{11} = \frac{14!}{(14-11)!} = \frac{14!}{3!}$

2) 8 students have to be arranged in 8 ranks, in how many ways can this be done

Answer: ${}^{8}P_{8} = 8!$

3) 8 students have to be arranged in six ranks, in how many ways can this be done

Answer: ${}^{8}P_{6} = 8\times7\times6\times5\times4\times3$

4) 8 students should be arranged in 10 ranks

Answer: It does not matter if the number of ranks is more than the number of persons. Approach will be same. Here we select the 8 ranks and then arrange the 8 persons in those ranks so the number of ways is ${}^{10}P_{8}$

5) 4 boys & 4 girls have to be seated on 8 chairs. In how many ways can you do the same , if boys & girls sit alternately

Answer: $4!\times4!\times2$

6) 4 boys & 4 girls have to be seated on 8 chairs. In how many ways can you do the same , if no 2 boys sit together

One way is to make the girls sit and then check where can the boys sit.

There will be 5 possible choices for the boys as can be seen below

__G__G__G__G__

So boys can select 4 seats out of 5 in 5 ways

Then the boys can arrange themselves in 4! Ways and the girls can arrange themselves in 4! Ways. So the total number of ways:

_ G _ G _ G _ G _ = $5 \times 4! \times 4!$ ways

OR

Lets number the chairs as 1 to 8

So boys can select the following set of chairs:

1,3,5,7

1,3,5,8

1,3,6,8

1,4,6,8

2,4,6,8

So there are 5 cases

In each case boys can arrange themselves in 4! Ways and girls can arrange themselves in 4! Ways

So total ways = 5 x 4! X 4!

7) 4 boys & 4 girls have to be seated on 8 chairs. In how many ways can you do the same , if a particular boy P wants to sit beside a particular girl R.

Answer: When 2 persons are to be adjacent to each other we normally take them as one unit. Say X.

So we have to arrange 3 boys, 3 girls and X

So 7 persons can be arranged in 7! Ways

Now in each case, the boy and the girl, P and Q can arrange themselves in 2!=2 ways

So total ways is: 7! X 2

8) 4 boys & 4 girls have to be seated on 8 chairs. In how many ways can you do the same , if a particular boy P , wants to sit between 2 girls R & S.

Answer: Let us take P, R and S as one unit, say X

Except these 3 there are 5 more persons, boys and girls put together

So there are in total 6 entities, 5 persons and X

They can be arranged in 6! Ways

Now in each case, P, Q and R can arrange themselves in 2! Ways

P will always sit in the middle of the 3 seats where as R and S can exchange their seats

So total ways = 6! X 2!

9) 4 boys & 4 girls have to be seated on 8 chairs. In how many ways can you do the same , if two girls P & Q do not sit together

Answer: We have already seen that the number of ways in which 2 persons can sit together is 7! X 2.

We also know that total arrangements possible are 8!

So we can say the number of ways in which a particular person will never sit with another person is: 8! – (7! X 2) = 7! (8 – 2) = 7! X 6

OR

Lets select 2 chairs for P and Q such that theya re not adjacent

So possible cases:

1,3-8 (6 cases)

2,4-8 (5 cases)

3,5-8 (4 cases)

4,6-8 (3 cases)

5,7-8 (2 cases)

6,8 (1 case)

So there can be 21 ways of selecting 2 chairs for P and Q

P and Q can arrange themselves in 2!=2 ways and rest can do it in 6! Ways

So total ways = 21 x 2 x 6! = 7x3x2x6!= 7! X 6

Circular Arrangement:

First we must understand why do we need to consider circular arrangement differently and why not take it same as linear or straight line arrangement

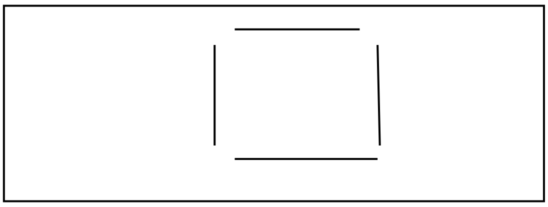
The only difference in circular arrangement is that there is no sense of direction in a circle, there are no starting or ending points as it is a closed loop, there are no looses ends.

In a linear arrangement even if the chairs are identical, exact replica of each other in terms of shape, design, colour etc., they can be considered distinct by virtue of their position. For example:

________ ________ ________ __________

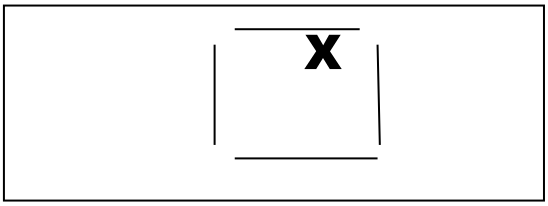
Here we can see that the chairs are identical but visually we can differentiate between them. We can identify them individually, because of their position.

In a circular arrangement this is not possible. For example

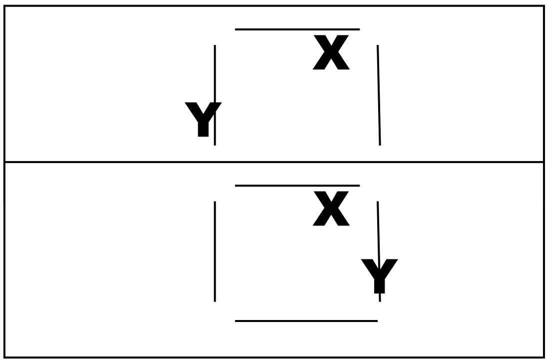


As can be seen, all the chairs and positions look identical.

This leads us to counting the same cases multiple times, if we use the simple formulae of n!. To solve this problem, we differentiate one chair by making any one person sit in a chair. It does not matter who sits as ultimately all of them will occupy different chairs. Once a person is seated in a chair, the arrangement is as good as a straight line. For example:



As we can see one chair is occupied by a person X. Now the chair to his right can be identified as the 1^{st} chair to his right, there will be a chair to his immediate left and a chair right opposite him. So each chair has a distinct identity with respect to X and can be differentiated. For example:



As we can see, Y sitting to the left of X or Y sitting to the right of X can be identified as different cases, which was not possible earlier.

So if there were n chairs and n persons, after making a person sit, there are (n-1) chairs and (n-1) persons who can now be seated in (n-1)! Ways.

Example: There are 10 persons and 7 chairs in a circle. In how amhy ways can they arrange themselves?

Select 7 out of 10 in $^{10}C_7$ ways = 120 ways. In each selection, the selected persons can arrange themselves in 6! Ways. So total ways = 120 x 6!

Garland Arrangement

In case of objects which can be turned upside down, like garlands, necklaces, or a giant wheel in a fair, the number of arrangements will be $\frac{(n-1)!}{2}$ ways

This is because the clock wise and anti clock wise arrangements there are same, as you can turn thing upside down, which we cant do with say a group a of person sitting in a circular arrangement (neither should we try!!!)

Example: They are 6 jasmine flowers and 4 rose flowers. Each flower is of different size. In how many ways 2 garlands consisting of the same flowers can be made if all the flowers are to be used?

Rose flower garland : $\frac{3!}{2}$ = 3

Jasmine flowers garland: $\frac{5!}{2}$ = 60

Total ways = 3 x 60= 180

One chair distinct

So, from above we can summarize that circular arrangement is meted out special treatment because all positions are identical and therefore we make a person sit and give each chair a distinct identity. So the question that comes to mind is what is one chair is already distinct, say a circular arrangement where all chairs are identical except once chair which is of different color. Since one chair is distinct, therefore in respect to it all chairs are distinct and therefore the answer will be (n!) and not (n-1)!

For example:

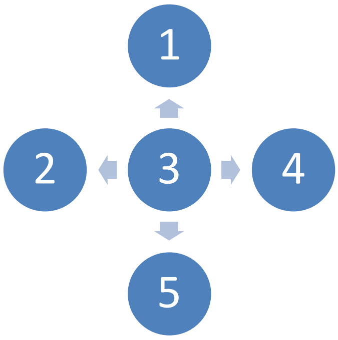
1) The court of a king is arranged in a circular fashion. King sits on a lavish throne and the 8 ministers sit on identical chairs. In how many ways can they arrange themselves:

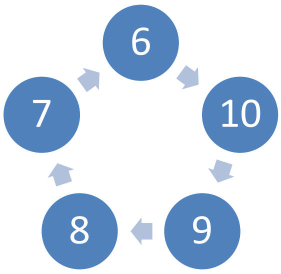
King will occupy his throne. The moment a person is seated on a chair, or if a chair is distinct then the arrangement is similar to a straight line arrangement. So number of ways = 8!

2) In a board meeting there are 6 members with chairs arranged in a circular fashion. All chairs are identical, however one chair is tagged as "Chairman". However board members do not care about such frivolities and anyone can occupy any chair. In how many different ways can they arrange themselves?

Here, since one chair is distinct therefore all chairs are distinct. So number of ways = 5!= 120

3) The seating arrangement in an office with 10 employees is as given below:





In how many ways can they arrange themselves?

In the above diagram, the is a cross shaped arrangement followed by a circular arrangement. However the position of chair 6is such that it has a distinct identity than all other chairs in the circle. If one chair is distinct all chairs are distinct, therefore number of seating arrangements = 10!

Solved Examples:

1) 4 boys & 3 girls have to be seated in 8 chairs in a circle. How many ways can this be done?

Answer: (n-1)!= (7-1)!=6!

2) 4 boys & 3 girls have to be seated in 8 chairs in a circle. How many ways can this be done If the chairs are of different colours

Answer: since the chairs are of different colours therefore each chair has a distinct identity, therefore the answer will be same as in a straight line: 7!

3) 3 boys & 3 girls have to be seated in 8 chairs in a circle. How many ways can this be done if the Boys & girls sit alternately

Answer: Let one boy sit. Now there are 5 vacant chairs. 3 girls and 2 boys are to be seated. There must be a girl on the either side of the boy who is seated. If the chair to the left of the seated boy is 1, then chair to his right will be 5. So girls can sit in chairs 1, 3 and 5 and the 2 boys will sit in chairs 2 and 4. So the answer will be 3! X 2!= 6 x 2= 12

4) 4 boys & 3 girls have to be seated in 7 chairs in a circle. How many ways can this be done if no 2 boys are sitting together

Answer: this is impossible. So 0 ways

5) 4 boys & 3 girls have to be seated in 7 chairs in a circle. How many ways can this be done if no 2 girls are sitting together
   Answer: Make one boy sit in 1 chair. Then there are 6 chairs left with 3 girls and 3 bous to be seated. Let the chair to the left of the seated boy is 1, and chair to his right will be 6 So the 3 girls can take 3 chairs in the following ways:
   1,3,5
   1,3,6
   1,4,6
   2,4,6
   In each of these cases, the 3 girls can arrange themselves in 3! Or 6 ways and the 3 boys can arrange themselves in 3! Or 6 ways. So total number of ways = Number of cases x number of arrangements of girls x Number of arrangements of boys = 4 x 6 x 6= 144
   We can also do this by making 1 girl sit in the chair. So now there will be 6 vacant chairs and 2 girls and 4 boys to be seated.
   The 2 girls cannot take chairs 1 aor 6 as they will become adjacent to the girl who is already seated. So the 2 girls can take 2 non adjacent chairs in the following ways:
   2,4
   2,5
   3,5
   In each of these cases, the 2 girls can arrange themselves in 2! Or 2 ways and the 4 boys can arrange themselves in 4! Or 24 ways. So total number of ways = Number of cases x number of arrangements of girls x Number of arrangements of boys = 3 x 2 x 24= 144
   .

6) 4 boys & 3 girls have to be seated in 7 chairs in a circle. How many ways can this be done if a particular boy 'A' wants to sit beside a particular girl B
   Answer: Make the boy and the girl sit. They can arrange themselves in 2!=2 ways. The remaining can arrange themselves in 5!=120 ways. So total number of arrangements = 2 x 120= 240
   Here, the couple is the reference point and once they are seated every chair gets a distinct identity in respect to them

7) 4 boys & 3 girls have to be seated in 7 chairs in a circle. How many ways can this be done if a particular boy 'A' does not want to sit with a girl B
   Answer: Total number of arrangement of 7 persons= 6!= 720
   Number of arrangements where A and B sit together (previous example) = 240
   Number of arrangements where A and B don’t sit together =720 -240= 480

8) 4 boys & 3 girls have to be seated in 7 chairs in a circle. How many ways can this be done if a particular boy 'A' wants to sit between 2 girls B & C
Answer: Make the 3 persons sit with A between B and C
B and C can arrange themselves in 2!=2 ways
Remaining 4 persons can arrange themselves in 4!=24 ways
Total arrangements = 2 x 24= 48