Comparison of Ratio

I) Equalization of either the Numerator or Denominator

a) If numerators are same then bigger the denominator, smaller is the fraction

i.e $\frac{8}{11} > \frac{8}{13} > \frac{8}{15} > \frac{8}{17}$

b) If denominators are same then bigger the numerator bigger is the fraction

i.e $\frac{11}{13} > \frac{9}{13} > \frac{7}{13} > \frac{5}{13}$

Which of the following is bigger? : $\frac{93}{169}\ and\ \frac{97}{147}$

In the 2nd fraction, numerator is bigger than the 1st fraction and denominator is less than 2nd fraction so 2nd fraction is definitely more than the 1st fraction.

Sometimes we try and identify whether one Numerator is a multiple of another Numerator or one Denominator is a multiple of another Denominator. And if so equalise the two.

Which of the following fractions is the largest? : 22/25, 11/13,

The first fraction has a Numerator 22 and the second one has a Numerator 11. Thus the second fraction can also be expressed as 22/26 (11/13). Now that we have equated the Numerators, compare the denominators. Since 25 < 26, therefore 22/25 > 11/13.

Which of the following fractions is the largest? : 7/13, 15/26,

Equate the denominators of the fractions. Therefore 7/13 = 14/26 < 15/26.

Therefore 7/13< 15/26

II) Identify fractions having same difference between Numerator and Denominator

In such cases, the following rule may be helpful:

i) If the value of the fractions is less than one, then higher the numerator, higher is the value of the fraction and smaller the numerator, smaller is the value of the fractions

ii) If the value of the fractions is more than one, then higher the numerator, smaller is the value of the fraction and smaller the numerator, higher is the value of the fractions"

Arrange the fractions in descending order $\frac{13}{19}, \frac{33}{39}, \frac{5}{11}, \frac{19}{25}, \frac{77}{83}, \frac{43}{49}, \frac{59}{65}$

All the fractions are less than one and in each case the difference between numerator and denominator is 6

Using rule (i) above, descending order or highest to lowest will be: $\frac{77}{83}, \frac{59}{65}, \frac{43}{49}, \frac{33}{39}, \frac{19}{25}, \frac{13}{19}, \frac{5}{11}$

Arrange the fractions in descending order $\frac{41}{32}, \frac{117}{108}, \frac{52}{43}, \frac{27}{18}, \frac{86}{77}, \frac{13}{4}, \frac{98}{89}$

All the fractions are more than one and in each case the difference between numerator and denominator is 8

Using rule (ii) above, descending order or highest to lowest will be: $\frac{13}{4}, \frac{27}{18}, \frac{41}{32}, \frac{52}{43}, \frac{86}{77}, \frac{98}{89}, \frac{117}{108}$

III) Rules

1) $\frac{a}{b} = \frac{c+am}{d+bm}$ if and only if $\frac{c}{d} = \frac{a}{b}$

i.e. $\frac{30}{40} = \frac{30+3m}{40+4m} = \frac{33}{44} = \frac{36}{48}$…. Etc. at *m* = 1,2,…..

Also, $\frac{a+c}{b+d} > \frac{a}{b}$ if $\frac{c}{d} > \frac{a}{b}$

Also, $\frac{a+c}{b+d} > \frac{a}{b}$ if $\frac{c}{d} < \frac{a}{b}$

This property is very useful when we compare two fractions

e.g., to compare between $\frac{10}{20}$ and $\frac{12}{22}$; $\frac{12}{22} = \frac{10+2}{20+2}$

Lets take a= 10, b=20

12- 10= 2 and 22-20=2

Therefore c=2 and d=2

Now, $\frac{c}{d} = \frac{2}{2} = 1\ and\ \frac{a}{b} = \frac{10}{20} < 1; therefore\ \frac{a}{b} < \frac{c}{d}$

So $\frac{a+c}{b+d} > \frac{a}{b}$

OR $\frac{12}{22} > \frac{10}{20}$

2) $\frac{a+k}{b+k} < \frac{a}{b}$ if for every positive $k, \frac{a}{b} > 1$

And, $\frac{a-k}{b-k} > \frac{a}{b}$

$\frac{a+k}{b+k} > \frac{a}{b}$ if for every positive $k, \frac{a}{b} < 1$ and $\frac{a-k}{b-k} < \frac{a}{b}$

Consider any ratio $\frac{a}{b}$. Now *x* is added to the numerator and the denominator of this fraction. Which of the following is greater : $\frac{a+x}{b+x}\ or\ \frac{a}{b}$?

It depends upon two factors:

i. If the ratio is proper or improper
ii. *x* is positive or negative.

$If\ \frac{a}{b} > 1\ and\ x > 0, or, \frac{a}{b} < 1\ and\ x < 0$

$\frac{a}{b} > \frac{a+x}{b+x}$

$$and\ if\ \frac{a}{b} > 1\ and\ x < 0, or, \frac{a}{b} < 1\ and\ x > 0$$

$$\frac{a}{b} < \frac{a+x}{b+x}$$

IV) Cross multiplication method

E.g., Let us compare 11/15 and 13/18.

$$\frac{11}{15} \times \frac{13}{18}$$

Cross multiplying numerator of 1st fraction with the denominator of 2nd fraction and denominator of 1st fraction with the numerator of 2nd fraction,

11 x 18	13 x 15
198	195

Since, 198 is greater than 195 the 1st fraction (11/15) is greater than the 2nd fraction (13/18).

V) Percentage comparison

Let us first understand this with the help of the following ratios

1st Case $\frac{10}{15} \xrightarrow[100\%]{} \xrightarrow{100\%} \frac{20}{30}$

2nd Case $\frac{10}{15} \xrightarrow[100\%]{} \xrightarrow{200\%} \frac{30}{30}$

3rd Case $\frac{10}{15} \xrightarrow[300\%]{} \xrightarrow{200\%} \frac{30}{60}$

In the 1st case, percentage change in numerator (100%↑) = percentage change in denominator (100%↑), so ratios are equal.

In the 2nd case, percentage change in the numerator (200%↑) > percentage change in the denominator (100%↑), so the 2nd ratio is greater than the 1st ratio.

In the 3rd case, percentage change in the numerator (200%↑) < percentage change in the denominator (300%↑), so the 1st ratio is greater than the 2nd ratio.

This particular example can also be seen as a general rule for determining the order of ratios.

Another method of comparing fractions is by comparing the percentage changes in denominators and numerators. The important points to remember are that when two fractions are compared, if the percentage increase in the numerator is more than the percentage increase in the denominator (where the first fraction is taken as reference), then the second

fraction is greater than the first fraction. Conversely, if the percentage increase in the denominator is greater than that in the numerator, then the second fraction is smaller than the first.

This is illustrated using the following examples.

(i) Compare $\frac{535}{426}$ and $\frac{589}{462}$

Increase in numerator is slightly greater than 10%, because 10% of 535 = 535 $\Rightarrow$ 535 + 53.5 = 588.5. Increase in denominator is slightly less than 10%, because 10% of 426 = 42.6 $\Rightarrow$ 426 + 42.6 = 468.6. Hence percentage increase in numerator is more than that in the denominator.

$\therefore$ The second fraction is greater.

$\Rightarrow \frac{589}{462} > \frac{535}{426}$

(ii) Compare $\frac{827}{1742}$ and $\frac{1103}{2439}$.

The percentage increase in the numerator is about $33^1/_3$% and the percentage increase in the denominator is about 40%. Hence the percentage increase in the denominator is more than that in the numerator.

$\therefore$ The first fraction is greater.

$\Rightarrow \frac{827}{1742} > \frac{1103}{2439}$

There are two types of mixtures of milk and water. In the first mixture, out of 12 litres of mixture, 5 litre is milk only and in the second mixture, 6 litre is milk and 12 litre is water. Which mixture is better in terms of milk's strength?

Ratio of milk and total in 1st and solution mixtures respectively : $\frac{5}{12}$ and $\frac{6}{18}$

Cross Multiplication : 5x18 = 90 and 6x12= 72. Therefore 1st ratio is bigger

Decimal: 5/12= .416 and 6/18= .333. So 1st ratio is bigger

Percentage: 1/12=8.33 % so 5/12 = 41.66% (approx.); 6/18= 33.33%. So 1st ratio is bigger

Using the rules, a= 5 and b=12, c= (6-5)=1 and d= (18-12)=6

So, $\frac{a}{b} = \frac{5}{12}$ and $\frac{c}{d} = \frac{1}{6} = \frac{2}{12}$

Therefore $\frac{a}{b} > \frac{c}{d}$ and $\frac{a}{b} > \frac{a+c}{b+d}$

Therefore $\frac{5}{12} > \frac{6}{18}$