CIRCLE

A circle is a set of points on a plane which lie at a fixed distance from a fixed point. There is one and only one circle passes through three non-collinear points.

Basic Terms

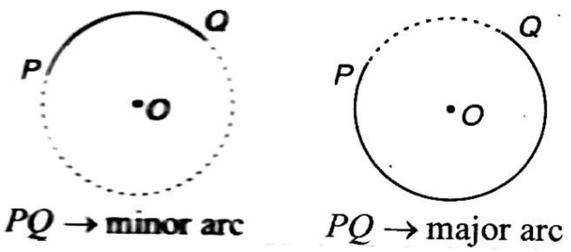
Radius: The fixed distance of the centre of a circle to any point on the boundary is called a radius.

Diameter: If the radius is extended to touch the circle on the other side it is called the diameter of the circle. The length of the diameter is twice the length of the radius. Diameter subtends an angle of 90^o on the circumference.

Circumference: The circumference of a circle is the distance around a circle, its boundary, and is equal to $2\pi r$.

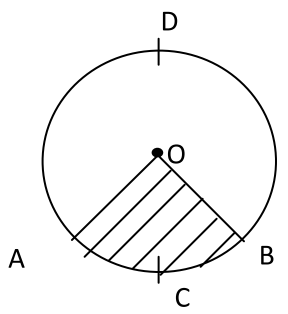
Arc: Any two points on the circle divides the circle into two parts the smaller part is called as minor arc and the larger part is called as major are.

It is denoted as '⌒'. In the given diagram $\widehat{PQ}$ is arc.



SEGMENT: The area formed between an arc and a chord is called a segment. Segment can be major or minor

SECTOR: Area between 2 radii & an arc is called a sector. Sectors can be major minor

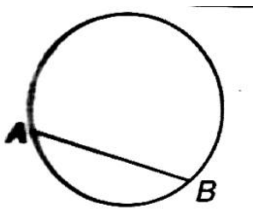


AOBC = Minor Sector

AOBD = Major Sector

Chords

A line segment whose end points lie on the circle. In the given diagram *AB* is a chord.

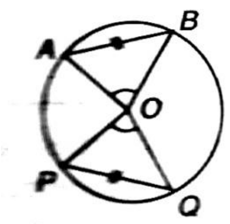


A chord divides a circle into two regions. These two regions are called the segments of a circle.

(a) major segment (b) minor segment.

Theorems related to chords

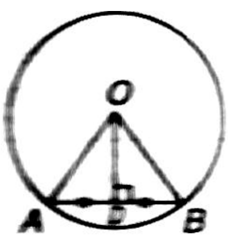
1) Equal chords subtend equal angles at the centre. (Inverse of this theorem is also true)



$\widehat{PQ} = \widehat{RQ}$ (or PQ = AB)

$\angle POQ = \angle AOB$

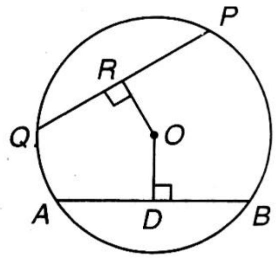
2) The perpendicular from the centre of a circle to a chord bisects the chord and conversely the bisecting line drawn on the chord from the centre is perpendicular to the chord. This means that perpendicular bisector of the chord will always pass through the centre.



a) if $OD \perp AB$, then $AB = 2AD = 2BD$

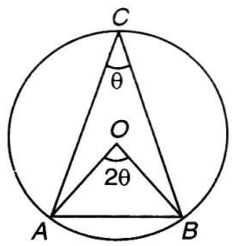
b) if D is the mid point of the chord AB then OD ⊥AB
c) If a ⊥ is drawn from D, which is the mid point of AB then it will pass through O

3) Length of a chord is inversely proportional to the distance from the centre and hence 2 chords equidistant from the centre will be equal in length and as corollary equal chords of a circle are equidistant from the centre.



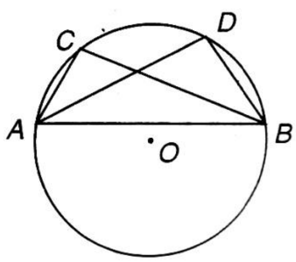
a) If AB= PQ then OD =OR
b) If OD =OR, then AB= PQ

4) The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. m∠AOB = 2m∠ACB

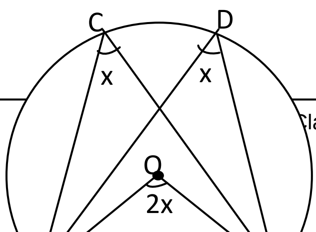


5) Angles in the same segment of a circle are equal

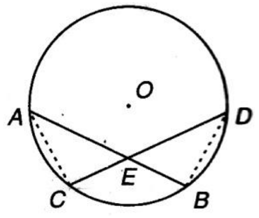
i.e., *∠ACB=∠ADB*

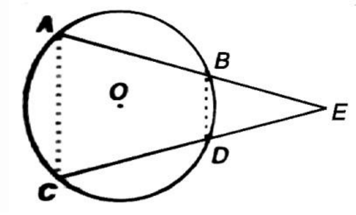


6) Angle subtended by a chord on the opposite arc will be supplementary to the original angle formed on the other arc.
To summarize the last 3 points:



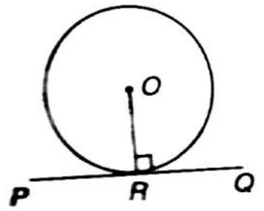
7) If two chords *AB* and *CD* of a circle, intersect inside a circle (outside the circle when produced at a point E), then *AE* x *BE* = *CE* x *DE*





Tangent

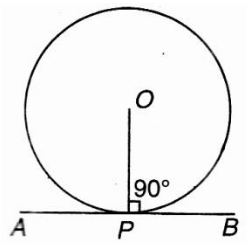
A line segment which has one common point with the circumference of a circle i.e., it touches only at only one point is called as tangent of circle. The common point is called as point of contact. In the given diagram *PQ* is a tangent which touches the circle at a point *R*.



Theorems about Tangents

1) A tangent at any point of a circle is perpendicular to the radius through the point of contact.

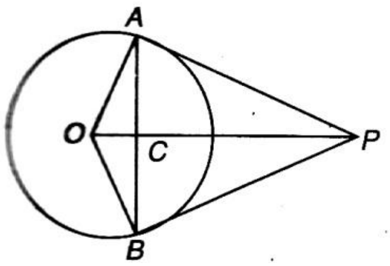
(Inverse of this theorem is also true)



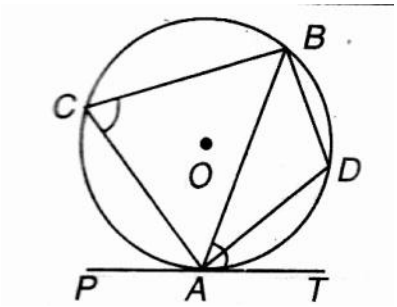
2) From an external point from which the tangents are drawn to the circle with centre O, then
a) The lengths of two tangents are equal i.e., *AP = BP*
b) They subtend equal angles at the centre: $\angle AOP = \angle BOP$

c) They are equally inclined to the line segment joining the centre of that point: $\angle APO = \angle BPO$

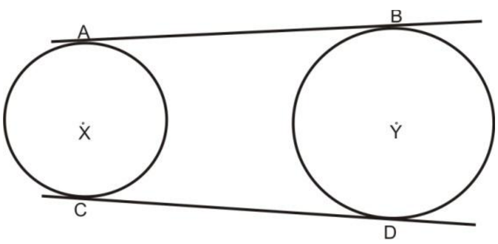
d) OP is the perpendicular bisector of AB: $OP \perp AB$ and $AC = BC$



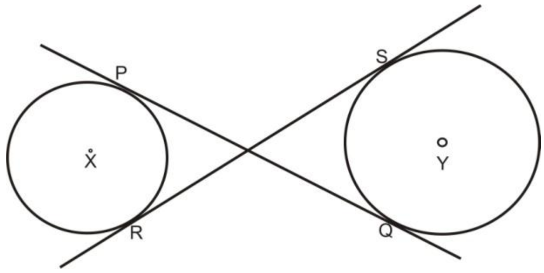
3) Alternate segment theorem : If from the point of contact of a tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram.: $\angle BAT = \angle BCA$ and $\angle BPA = \angle BDA$



4) For the two circles with centre X and Y and radii r_1 and r_2. AB and CD are two Direct Common Tangents (DCT), then the length of DCT = $\sqrt{(distance\ between\ centres)^2 - (r_1 - r_2)^2}$



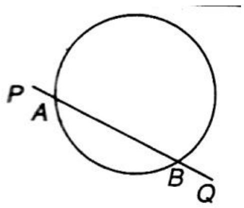
5) For the two circles with centre X and Y and radii r_1 and r_2. PQ and RS are two transverse common tangent, then length of TCT= $\sqrt{(distance\ between\ centres)^2 - (r_1 + r_2)^2}$



Secant

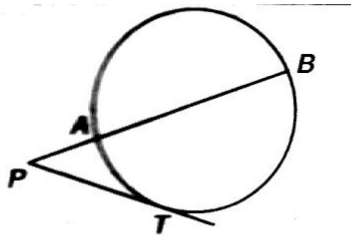
A line segment which intersects the circle in two distinct points, is called as secant.

In the given diagram secant *PQ* intersects circle at two points at *A* and *B*.



Theorems about Secant

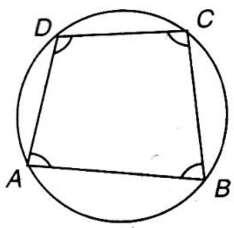
1) If PB be a secant which intersects the circle at *A* and *B* and *PT* be a tangent at *T* then *PA.PB* = $(PT)^2$



Cyclic Quadrilateral

A quadrilateral whose all the four vertices lie on the circle.

The sum of pair of opposite angles of a cyclic quadrilateral is 180°

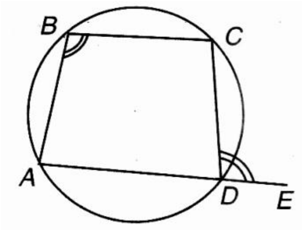


$\angle DAB + \angle BCD = 180°$

$\angle ABC + \angle CDA = 180°$

(Inverse of this theorem is also true)

If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle



$m\angle CDE = m\angle ABC$

2 CIRCLES

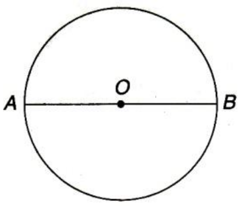
Two circles touching externally will have a distance between its centres which will be the sum of their respective radii

Two circles touching internally will have a distance between its centres which will be the difference between their respective radii

CIRCLES Mensuration

1. Area of a circle = πR^2 ($R \rightarrow$ radius of the circle)
2. Circumference of the circle = $2\pi R$
3. Length of an arc = $2\pi R\left(\frac{\theta}{360°}\right)$
4. Area of a sector = $\pi R^2\left(\frac{\theta}{360°}\right) = \frac{1}{2}(\text{arc} \times R)$
5. Area of segment = $\pi R\left(\frac{\theta}{360°}\right) = \frac{R^2}{2}\sin\theta$

Diameter = 2 × radius



OB = OA→ radius

AB→ diameter

Solved Examples:

1) The radius of a circular wheel is $5\frac{1}{4}$m. How manyrevolutions will it make in travelling 33 km?

Total distance (travelled) = 33 km = 33000 m

Distance travelled in one revolution

= circumference of the wheel

= $2\times \pi \times r = 2 \times \frac{22}{7} \times \frac{21}{4} = 33m$

∴number of revolutions in 11km = $\frac{33000}{33}$

= 1000 revolution

2) A circular road runs round a circular garden. If the difference between the circumference of the outer circle and the inner circle is 88m. Find the width of the road.

Let R and r be radii of outer circle and inner circle respectively

∴ Width of the road = R-r

∴ $2\pi r - 2\pi r = 88m$

$$\Rightarrow 2\pi(R - r) = 88m$$

$$\Rightarrow (R - r) = 14m \left(\because \pi = \frac{22}{7}\right)$$

3) The radius of a circle is 6 m. What is the radius of another circle whose area is 36 times that of the first?

Ratio of areas $=$ (ratio of radii)^2

$\frac{36}{1} = (\text{ratio of radii})^2$

$\Rightarrow$ratio of radii $= \frac{6}{1}$

Therefore radius of another circle is 6 times

Hence the required radius = 36m

4) What is the radius of a circle whose area is equal to the sum of the areas of two circles whose radii are 20 cm& 21 cm respectively.

$$\pi R^2 = \pi r_1^2 + \pi r_2^2$$
$$\pi R^2 = \pi(r_1^2 + r_2^2)$$

$R^2 = 841$

$R = 29cmv$

5) In a circle of radius 21 cm, an arc subtends an angle of 120° at the centre.

(a) Find the area of the sector.

(b) Find the length of the arc

(a) Area of the sector = $\pi R^2 \left(\frac{?}{360^\circ}\right)$

$= \frac{22}{7} \times 21 \times 21 \times \frac{120º}{360º}$

$= 22 \times 3 \times 21 \times \frac{1}{3}$

$= 462cm$

(b) Length of the arc= $2\pi r \left(\frac{?}{360^\circ}\right)$

$= 2 \times \frac{22}{7} \times 21 \times \frac{120º}{360º} = \mathbf{44cm}$